

Math 4740 - Homework # 3
Conditional Probability, Law of Total Probability,
Independence

Part 1 - Independent events

1. A 6-sided die is rolled twice. Let A denote the event that the sum of the outcomes is odd and B denote the event that the die lands on a 2 on the first toss. Are A and B independent events? Why or why not?
 2. Suppose you flip a coin three times. Let E be the event that the first flip is a heads H. Let F be the event that the second flip is a tails T. Are E and F independent? Show why or why not using the formulas from class.
 3. Suppose you play the following game. First, you roll a 4-sided die. Then you flip a coin. Further suppose that the 4-sided die is weighted so that a 1 occurs about once in every six rolls, a 2 occurs about once in every six rolls, a 3 occurs about once in every six rolls, and a 4 occurs about three times in every six rolls.
 - (a) Draw the probability tree for this game.
 - (b) Let A be the event that you get a 1 on the die. Let B be the event that you get heads on the coin. Are A and B independent in your probability space?
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Part 2 - Law of total probability

4. Suppose that there are 2 bags. Bag 1 contains 3 white balls and 3 red balls. Bag 2 contains 4 white balls and 2 red balls. Suppose you do this experiment. You flip a coin. If tails comes up then you pick a ball from bag 1. If heads comes up then you pick a ball from bag 2.
 - (a) When you do the above experiment, what is the probability of picking a red ball?
 - (b) When you do the above experiment, what is the probability of picking a white ball?

5. Suppose you play the following game. First, you roll a 4-sided die. Then, you flip a coin. Further suppose that the 4-sided die is weighted so that a 1 occurs with probability $1/6$, a 2 occurs with probability $1/6$, a 3 occurs with probability $1/3$, and a 4 occurs with probability $1/3$. The coin is a usual kind of coin, that is the probability is $1/2$ for heads and $1/2$ for tails.
- (a) Draw the probability tree for this experiment.
 - (b) When you do the above experiment, what is the probability of rolling an even number and then getting a tails?
6. An bag contains 10 white and 12 red chips. One chip is drawn from the bag and discarded (ie it is not put back into the bag). Then a second chip is drawn from the bag and discarded. Then a third chip is drawn from the bag. What is the probability that the third chip drawn is red?
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Part 3 - Conditional probability

7. Suppose that two 6-sided dice have been thrown. Suppose you can't see the outcome of the dice, but someone tells you that the sum of the dice is divisible by 5. What is the probability that both of the dice have landed on 5's.
8. Suppose your friend flips a coin 3 times in a row. They don't show you the results. They tell you that at least 1 of the flips was a head. They ask you: What is the probability that at least 2 of the flips were heads?
9. Suppose that two cards are chosen randomly from a standard 52 card deck. Recall that the aces are $A\heartsuit$, $A\diamondsuit$, $A\clubsuit$, $A\spadesuit$
- Let B be the event that both cards chosen are aces.
- Let A be the event that at least one of the cards chosen is an ace.
- Let A_s be the event that one of the cards chosen is the ace of spades, ie $A\spadesuit$.
- (a) Compute $P(B|A_s)$
 - (b) Compute $P(B|A)$

10. From an ordinary 52 card deck, cards are drawn one by one, at random, and without replacing them back into the deck. What is the probability that the 4th heart is drawn on the 10th draw? For example, such a sequence could be as follows: $K\heartsuit, 9\clubsuit, 9\spadesuit, 4\diamondsuit, 5\heartsuit, 2\clubsuit, A\clubsuit, 8\heartsuit, 7\diamondsuit, A\heartsuit$.

[Hint: Let F denote the event that in the first nine draws there are exactly three hearts and E be the event that the tenth draw is a heart. Use $P(F \cap E) = P(F)P(E|F)$.]

Part 4 - This part is optional. We will use it for craps. I might not cover it in class until the end of the semester.

11. Suppose you roll two 6-sided dice over and over until either the sum of the dice is 6 or the sum of the dice is 7.
- (a) What is the probability that you roll a sum of 6 before you roll a sum of 7?
 - (b) What is the probability that you roll a sum of 7 before you roll a sum of 6?
12. Suppose you repeat the following experiment. You take a standard 52 card deck and randomly draw a card. If you get an ace or a face card then you stop the experiment, otherwise you put the card back into the deck and shuffle the deck. You continually repeat this experiment until either an ace card comes up or a face card comes up. [Recall that a face card means either a Jack, Queen, or King.]
- (a) What is the probability that an ace comes up before a face card?
 - (b) What is the probability that a face card comes up before an ace?

Part 5 - The following problems are optional to do if you want extra practice with more challenging conditional probability problems

13. Suppose that there are three cards. The first card is red on both sides. The second card is black on both sides. The third card is black on one side and red on the other side. The three cards are mixed up in a hat. Then one card is randomly drawn from the hat and put down on the ground. If the upper side of the chosen card is red, what is the probability that the other side is black?
14. A box contains 7 red and 13 blue balls. Two balls are selected at random and are discarded without their colors being seen. If a third ball is drawn randomly and observed to be red, what is the probability that both of the discarded balls were blue?